Rules for Logs

What is a Log?
(Or, more formally, what is the logarithm of a number?)

A log (base 10) of a number is a power you would put on 10 to equal that number.
In Base 10 (which we assume when there is no other number written),
the log is the exponent of 10 that would equal the number.

E.g. The log of 100 is 2 because 2 is the exponent of 10 that gives you 100.

$\text{Log}_{10}100 = 2$ or $\log_{10}10^2 = 2$ because $10^2 = 100$

The exponent of the number when it is expressed as a power of ten is the number's log!
If you can easily express the number as 10 to a power, you know the log- it's the power!

You can find logs to other bases:

$\text{Log}_2\, 8 = 3$ because $\text{Log}_2\, 2^3 = 3$ or $8 = 2^3$

In a logarithmic equation,

- the EXPONENT is alone on the right,
- the BASE is the number lower than the line , just right of 'log' (if there is no number, assume it is 10)
- the value you are finding the log of is just before the equal sign.

Why are logs important? Why do we have to learn them?

Before we had scientific calculators, logs were used to do calculations. Believe it or not, logs make calculations on large numbers easier than doing them by hand.

Logarithms are used in chemistry and soil science (pH levels), and in biomathematics and ecology (growth rates) and geology (Richter scale).

We still have to learn them because the rules of logarithms are still used in many areas of science. If you don't understand logs and how to use these log rules, you will not be able to understand those areas of science. For example, log rules are used to find the water infiltration in soils, in biomathematics and ecology to construct growth models from statistical data, and in many areas of science.

In the days before the introduction of scientific calculators (the 1960's and earlier), these rules were used by everyone to multiply large numbers and to find powers of numbers.

What are the Log Rules?

Because Logs are Exponents, we use the rules of exponents to manipulate logs.
Remember the rules of exponent:

$a^x \cdot a^y = a^{x+y}$

Since logs are exponents,

Product Rule: log (x times y) = log x + log y and vice versa: log x + log y = log xy
Quotient Rule: log (x / y) = log x - log y and vice versa: log x - log y = log x/y
Power Rule: $\log x^n = n \log x$ and vice versa: $n \log x = \log x^n$

NOTE: <u>These only work if the logs all have the same base. If they don't there is no way to combine them.</u>

<u>Examples</u>:

log 3 + log 4 = log 12
OR
Log 30 = log 5 + log 6

Log 10 = log 30/3 = log 30 – log 3

We can actually find these logs on the calculator and check that the equations are true.

But what if the base is NOT 10? We have no way to check them on the calculator, unless we use the CHANGE OF BASE Rule:

Change of Base Rule: $\log_x b = \log b / \log x$ (both base 10, which we <u>can</u> evaluate on calculator)

$\log_3 30 = \log_{10} 30 / \log_{10} 3 = (\log 10 + \log 3) / \log 3 = (\log 10 / \log 3) + 1$