VECTORS

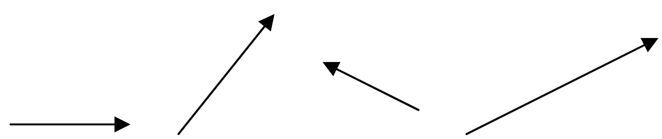
Definition: A ***vector*** is a quantity that has both magnitude and direction.

NOTE: The position of a vector has no bearing on its definition. A vector can be slid horizontally or vertically without change. Be careful, however, not to rotate it.

In print, vectors are traditionally denoted by boldface type. In writing, they are donated by an arrow:

$\vec{A}$ or $\bar{A}$ to be read vector A.

Vectors can be shown graphically



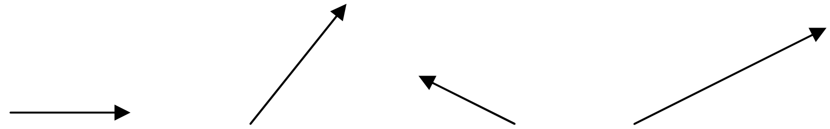
Or by direction

3 units at 30˚
4.2 units at (-27˚)
6.3 units at 150 ˚
12 units due north
4 units 5˚ east of south

The negative of vector A has the same magnitude and orientation, but the direction is 180˚ offset from the direction of vector A.

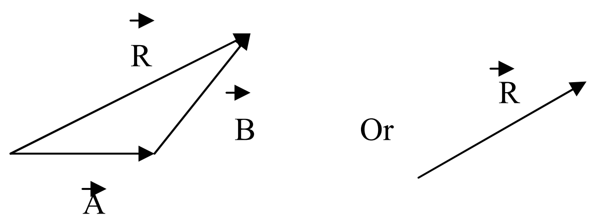
Vectors can be added or subtracted graphically or by the use of trigonometry. Scalar multiplication, that is, multiplication by a magnitude, can also be done graphically or by the use of trigonometry. The sum is called the resultant $\vec{R}$.

Examples:

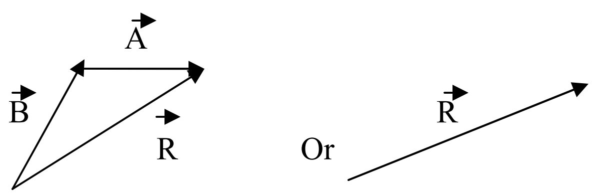


$\vec{A}$ $\vec{B}$ $\vec{C}$ $\vec{D}$

(1) $\vec{A}$ + $\vec{B}$ **:**

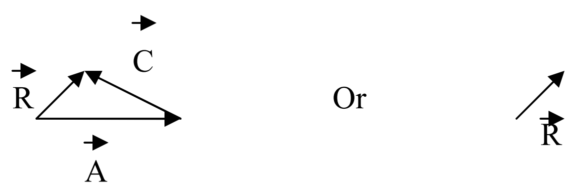


$\vec{B}$ + $\vec{A}$ **:**

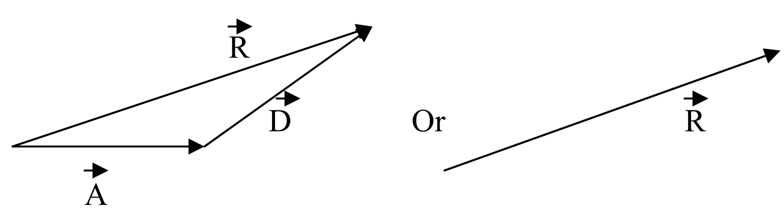


Please note that vector addition is commutative.

(2) $\vec{A}$ + $\vec{C}$

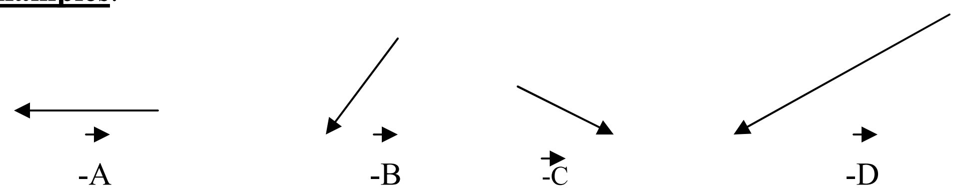


(3) $\vec{A}$ + $\vec{D}$

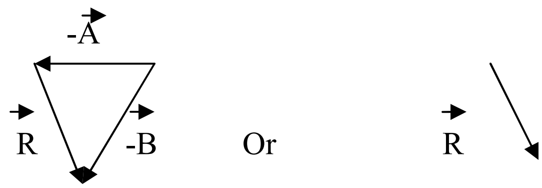


It is readily seen that the horizontal components of each of the addend vectors are added to give the horizontal component of the resultant vector; also, the vertical components of each of the addend vectors are added to give the vertical component of the resultant vector.

More Examples:



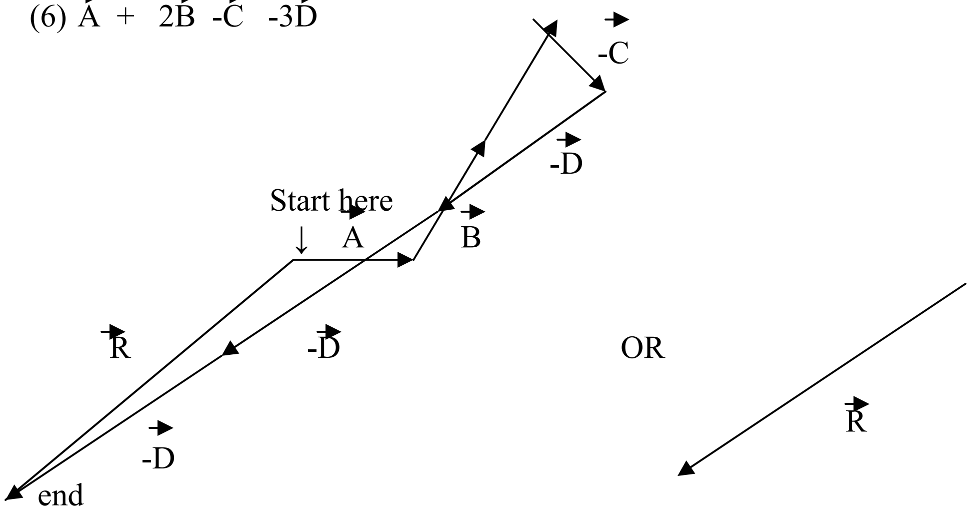
(4) $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$:



(5) $\vec{B} - \vec{A} = \vec{B} + (-\vec{A})$

Please note $\vec{R}$ from example (5) is the negative of $\vec{R}$ from example (4). This is not unexpected since **a – b = - (b – a)** in real numbers as well.

(6) $\vec{A} + 2\vec{B} - \vec{C} - 3\vec{D}$

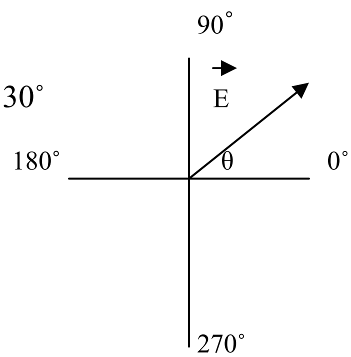


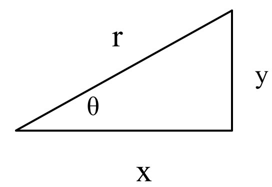
Adding, subtracting graphically is good when the vectors are given graphically. It works less well when the vectors are given in other forms.

To add or subtract using trigonometry, a vector must be broken into vertical and horizontal components. Place the vector in standard position.

Example:

(7) 3 units at 30°





Remember that

$$\cos\theta = \frac{x}{r}$$

$$\sin\theta = \frac{y}{r}$$

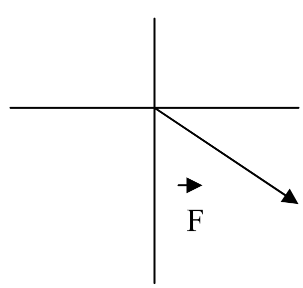
Therefore $x = r\cos\theta$ horizontal component
$y = r\sin\theta$ vertical component

$\vec{E}$ = 3 units at 30°

	H	**V**
$\vec{E}$ = 3 units at 30°	$3\cos 30°$	$3\sin 30°$
	$= 3\left(\frac{\sqrt{3}}{2}\right)$	$= 3\left(\frac{1}{2}\right) = \frac{3}{2}$ or 1.5
	$= \frac{3}{2}\sqrt{3}$ or 2.5981	

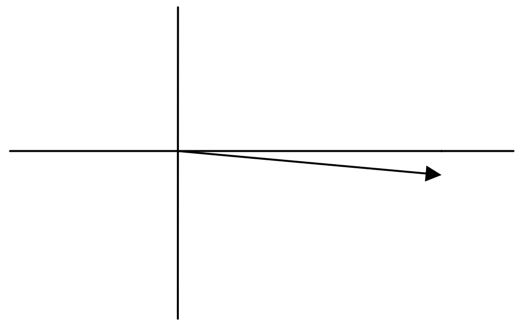
(8) $\vec{F}$ = 4.2 units at (-27°)

H	**V**
4.2 cos (-27°)	4.2 sin (-27°)
= 3.7422	-1.9068



(9) Let us add vector $\vec{E}$ and $\vec{F}$ from the previous example

	H	V
$\vec{E}$ 3 units at 30˚	2.5981	1.5000
$\vec{F}$ 4.2 units at (-27˚)	3.7422	-1.9068
	6.3403	-0.4068



The resultant vector has a horizontal component of magnitude 6.3403 and a vertical component of magnitude 0.4068 in the negative direction.

Just what is the actual magnitude of this new vector?

Since $x = r\cos\theta$ and $y = r\sin\theta$

We will have $x^2 + y^2 = r^2$

both by Pythagorus' theorem and by the trigonometric identity $\sin^2\theta + \cos^2\theta = 1$.

$$x^2 = r^2 \cos^2\theta \qquad y^2 = r^2\sin^2\theta$$

$$x^2 + y^2 = r^2 \cos^2\theta + r^2\sin^2\theta = r^2(\cos^2\theta + \sin^2\theta) = r^2$$

And since $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{r\sin\theta}{r\cos\theta} = \dfrac{y}{x}$

We have $\theta = \tan^{-1}\dfrac{y}{x}$

As the direction of our resultant vector.

(9) continued

$$r^2 = (6.3403)^2 + (-0.4068)^2$$

$$r = \sqrt{(6.3403)^2 + (-.4068)^2} = 6.353369\ldots.$$

$$\theta = \tan^{-1}\left(\frac{-0.4068}{6.3403}\right) = -3.6711\ldots.$$

Giving the answer in the same format as the original problem, we have

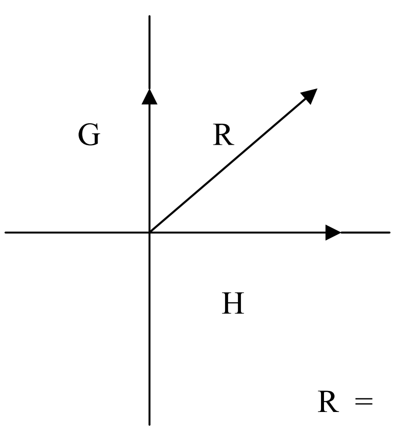
$$\vec{R} = 6.4 \text{ units at } (-4^\circ)$$

(10) $\vec{G}$ = 12 units due north

$\vec{H}$ = 16 units due east

If you are not accustomed to working in navigational directions, convert to standard position.

	H(x)	**V(y)**
$\vec{G}$ = 12 units at 90˚	$12 \cos 90^\circ = 0$	$12 \sin 90^\circ = 12$
H = 16 units at 0 ˚	$16 \cos 0^\circ = \underline{16}$	$16 \sin 0^\circ = \underline{0}$
	16	12



$$r^2 = x^2 + y^2 = 16^2 + 12^2 = 256 + 144 = 400$$

$$r = 20 \text{ units}$$

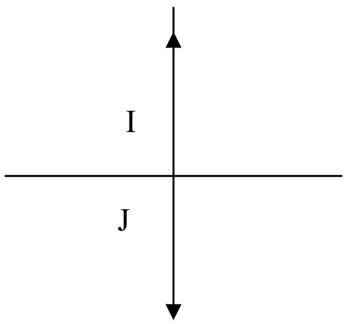
$$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{12}{16} = \tan\frac{3}{4} = 36.9^\circ$$

R = 20 units at

(11) I = 4 units at 90° — $4\cos 90^\circ = 0$ — $4\sin 90^\circ = 4$

J = units at 270° — $5\cos 270^\circ = 0$ — $5\sin 270^\circ = -5$

$\overline{}$ 0 — $\overline{}$ -1



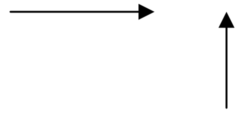
$r^2 = 02 + (-1)2 = 1$ — $r = 1$

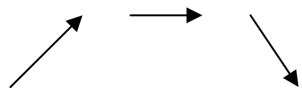
$\theta = \tan^{-1} \frac{y}{x} = \tan{-1} \frac{-1}{0}$ undefined neg.

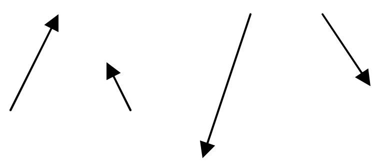
means 270°

PROBLEM SET I

(1) Graphically add the vectors:

(a) 

(b) 

(c) 

(2) Give the horizontal and vertical components of the following vectors:

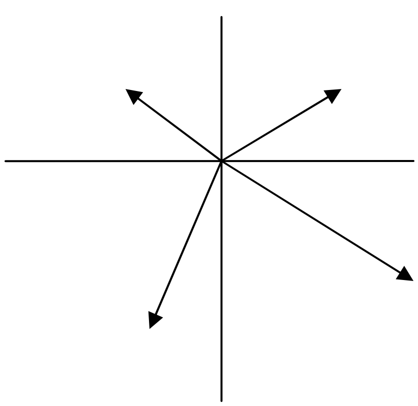
(a) A = 5 lb at 42°

(b) B = 20 m at 110°

(c) C = 13.9 N at 234°

(d) D = 62.3 $\frac{m}{s}$ at 315°

(3) Add the following vectors using the representation of the horizontal and vertical components on the appropriate axes.



(4) Add the following vectors by first finding the horizontal and vertical components. Give the resultant in the same format as the original information.

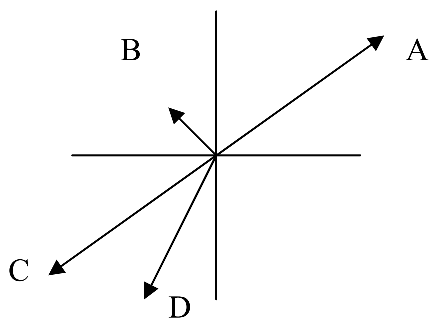
(a) 5 lb at 42°
3 lb at 57°

(b) 20 m at 110°
14 m at 32°

(c) 13.9 N at 234°
11.6 N at 56°

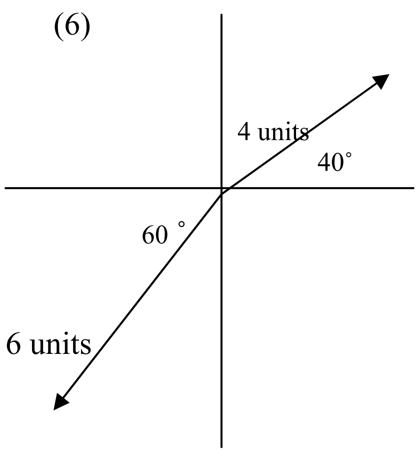
(d) 62.3 $\frac{m}{s}$ at 315°
47.8 $\frac{m}{s}$ at 37°

(5) Find the resultant vectors:



A: 5 units at 30°
B: 2 units at 135°
C: 8 units at 210°
D: 4 units at 240°

(6)

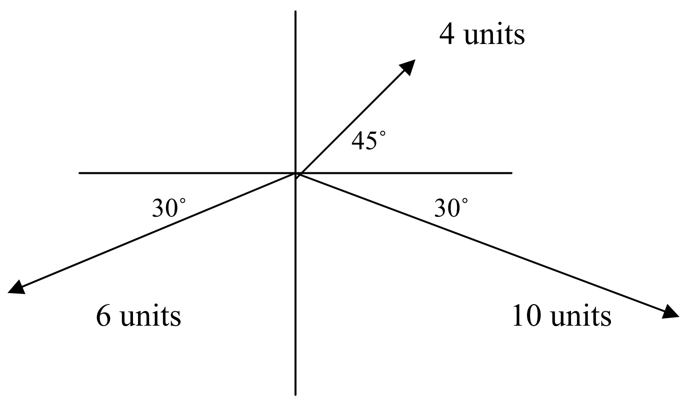


(7) 14 lb at 36°
20 lb at 75°
38 lb at 100°

(8) 6 lb at 30°
6 lb at (-30°)
7 lb at 180°

Find the resultant vectors:

(9)



(10) 85 N at (-13°)
126 N at 49°
72 N at 168°
46 N at 185°

PROBLEM SET II

Given horizontal components, R_x

And vertical components, R_y

Find the resultant $\vec{R}$

That is, find R, the magnitive, and θ, the angle of the resultant vector $\vec{R}$

1. $R_x = 4.23$ $R_y = 4.00$

2. $R_x = 82.9$ $R_y = 45.0$

3. $R_x = -.877$ $R_y = 4.25$

4. $R_x = -611$ $R_y = -838$

5. $R_x = -871$ $R_y = 7420$

6. $R_x = -44.9$ $R_y = -19.9$

7. $R_x = 0.742$ $R_y = -3.77$

8. $R_x = 5.79$ $R_y = -8.89$

Add the given vectors by first finding their horizontal and vertical components, then using the Pythagnean Theorem.

9. $A = 6.7 \quad \theta_A = 7.23^\circ$

$B = 44.9 \quad \theta_B = 74.2^\circ$

10. $A = 498 \quad \theta_A = 126^\circ$

$B = 889 \quad \theta_B = 40.0^\circ$

11. $A = 7.40 \quad \theta_A = 81.5^\circ$

$B = 4.23 \quad \theta_B = 732.6^\circ$

12. $A = 9.76 \quad \theta_A = 81.5^\circ$

$B = 1.76 \quad \theta_B = 278.4^\circ$

13. $A = 70.3 \quad \theta_A = 235.2^\circ$

$B = 88.9 \quad \theta_B = 85.8^\circ$

$C = 36.1 \quad \theta_C = 101.1^\circ$

14. $A = 5190 \quad \theta_A = 267.9^\circ$

$B = 2310 \quad \theta_B = 357.7^\circ$

$C = 670 \quad \theta_C = 28.7^\circ$

Solve the given problems:

15. Two forces, one of 54.2 lb and the other of 14.1 lb., act on the same object and a right angle to each other. Find the resultant of these forces.

16. Two forces, one of the 176 N and the other of 87 N, pull on an object. The angle between these forces is 25.0˚. What is the resultant of these forces?

17. A ship sails 63.2 km due north after leaving its pier. It then turns and sails 76.5 km east. What is the displacement of the ship from its pier?

18. A rocket is traveling at an angle of 85.8˚ with respect to the horizontal at a speed of 4920 km/h. What are the horizontal and vertical components of the velocity?